TIGA JAWABAN PENYELESAIAN OPTIMAL DALAM PROBLEM TRANSPORTASI DENGAN VAM AND MODI METHOD

Rudy Santosa Sudirga
Email : rudysudirga@yahoo.com

Penulis

Rudy Santosa Sudirga adalah staf pengajar di Program Studi Manajemen Universitas Bunda Mulia dengan bidang quantitative management.

Abstract

The famous method to determine and solve transportation problem is the transportation model VAM and MODI method, the Northwest-Corner and Stepping-Stone method, and the Assignment method. We see how to develop an initial solution to the transportation *problem with VAM (Vogel's Approximation Method) and MODI (Modified Distribution)*. VAM is not quite as simple as the Northwest Corner approach, but it facilitates a very good initial solution, as a matter of fact, one that is often the optimal solution.

VAM method tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative. This is something that Northwest Corner Rule does not do. To apply VAM, we first compute for each row and column the penalty faced if we should ship over the second-best route instead of the least-cost route. After the initial VAM solution has been found, you should evaluate it with either the Stepping-Stone method or the MODI method. The MODI (Modified Distribution) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths. Because of this, it can often provide considerable time savings over the Stepping-Stone method for solving transportation problems. If there is a negative index indicating an improvement can be made, then only one Stepping-Stone path must be

found. This is used as it was before to determine what changes should be made to obtain the improved solution.

In the Northwest-Corner rule, the largest possible allocation is made to the cell in the upper left-hand corner of the tableau, followed by allocations to adjacent feasible cells. While the Stepping-Stone method is an iterative technique for moving from an initial feasible solution to an optimal feasible solution, and continues until the optimal solution is reached. The Stepping-Stone path method is used to calculate improvement indices for the empty cells. Improved solutions are developed using a Stepping-Stone path.

The Assignment method, which is simple and faster to solve the transportation problem by reducing the numbers (cost) in the table/tableau until a series of zeros is found, or zero opportunity costs, which means that we will reach the optimal cost allocations. Once we have reached the optimal cost allocations, we then allocate each sources or supply according to some points of demand (destinations). Assignment Method is a specialized form of optimization linear programming model that attempts to assign limited capacity to various demand points in a way that minimizes costs.

The special cases of transportation problem included degeneracy (a condition that occurs when the number of occupied squares in any solution is less than the number of rows plus the number of columns minus 1 in a transportation table), unbalanced problems, and multiple optimal solutions. At this opportunity, we would like to demonstrate the multiple optimal solutions. We will see how the VAM and MODI method may be viewed as a special case of solving the multiple optimal solutions of the transportation problem.

Keywords

Transportation, VAM and MODI.

PENDAHULUAN

Render, B., Stair, R.M., Hanna, M.E. (2010:368), the transportation algorithm is an iterative procedure in which a solution to a transportation problem is found and evaluated using a special procedure to determine whether the solution is optimal. If it is optimal, the process stops. If it is not optimal, a new solution is generated. This new solution is at least as good as the previous one, and it is usually better. This new solution is then evaluated, and if it is not optimal, another solution is generated. The process continues until the optimal solution is found.

Render, B., Stair, R.M., Hanna, M.E. (2009:457), Stevenson W.J. (2009:743), and Taylor B.W. (2010:248,258), pemikiran dari tulisan atau artikel ini adalah berdasarkan teori bahwa problem transportasi dan problem assignment keduanya adalah termasuk kategori linear programming dan menggunakan teknis pemecahan secara linear programming pula. Ide dari tulisan atau artikel ini muncul pertama kali pada saat penulis memberikan kuliah manajemen sains beberapa waktu yang lalu, dan penulis mempunyai kesimpulan bahwa sebenarnya problem transportasi dengan multiple optimal solutions dapat juga diselesaikan dengan Assignment method, VAM and MODI dan Northwest Corner and Stepping-Stone. Pertama kali penulis mencoba menghitung hasilnya dengan Assignment method, lalu membandingkannya dengan VAM and MODI, dan Northwest-Corner and Stepping-Stone, dan setelah melakukan pengamatan beberapa kali dan juga melakukan tes penyelesaian dengan software-program POM-QM for Windows ternyata memberikan hasil yang sama baik dilihat dari alokasinya maupun total biaya optimalnya, sehingga penulis berkesimpulan bahwa penyelesaian problem transportasi dengan multiple optimal solutions dapat juga diselesaikan dengan Assignment method, VAM and MODI dan Northwest Corner and Stepping-Stone.

Pada tulisan ini penulis hanya akan membahas 3 (tiga) jawaban penyelesaian optimal dalam problem transportasi dengan VAM and MODI method.

Supranto, J. (2006:186,193), VAM (*Vogel's Approximation Method*), yang disingkat dengan VAM walaupun tidak selalu menghasilkan pemecahan optimum akan tetapi bisa juga memberikan pemecahan yang optimal. VAM tidak menjamin suatu penyelesaian yang optimum, akan tetapi sangat berguna karena alasan berikut ini: (1) sering menghasilkan pemecahan optimum, (2) dapat menghasilkan penyelesaian yang mendekati optimal dengan usaha yang tidak banyak, sehingga dapat dipergunakan untuk melangkah menuju ke pemecahan yang optimal.

Render, B., Stair, R.M., Hanna, M.E. (2009:446), VAM method tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative. This is something that Northwest Corner Rule does not do. To apply VAM, we first compute for each row and column the penalty faced if we should ship over the second-best route instead of the least-cost route. After the initial VAM solution has been found, you should evaluate it with either the Stepping-Stone method or the MODI method. The MODI (Modified Distribution) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths. Because of this, it can often provide considerable time savings over the Stepping-Stone method for solving transportation problems. If there is a negative index indicating an improvement can be made, then only one Stepping-Stone path must be found. This is used as it was before to determine what changes should be made to obtain the improved solution.

Render, B., Stair, R.M., Hanna, M.E. (2010:370, 372, 393), in the Northwest-Corner rule, the largest possible allocation is made to the cell in the upper left-hand corner of the tableau, followed by allocations to adjacent feasible cells. While the Stepping-Stone method is an iterative technique for moving from an initial feasible solution to an optimal feasible solution, and continues until the optimal solution is reached. The Stepping-Stone path method is used to calculate improvement indices for the empty cells. Improved solutions are developed using a Stepping-Stone path.

Render, B., Stair, R.M., Hanna, M.E. (2010:385), each assignment problem has associated with a table, or matrix. Generally, the rows contain the objects or people we wish to assign, and the columns comprise the tasks or things we want them assigned to. The numbers in the table are the cost associated with each particular assignment.

An assignment problem can be viewed as a transportation problem in which the capacity from each source (or person to be assigned) is 1 and the demand at each destination (or job to be done) is 1. Such formulation could be solved using the transportation algorithm, but it would have a severe degeneracy problem. However, this type of problem is very easy to solve using the assignment method.

Winston, W.L. (2004:378), the Northwest-Corner method does not utilize shipping costs, so it can yield an initial basic feasible solution that has a very high shipping cost. Then determining an optimal solution may require several pivots. The Minimum-Cost method uses the shipping costs in an effort to produce a basic feasible solution that has a lower *total cost. Hopefully, fewer pivots wil then be required to find the problem's optimal* solution. Because the Minimum-Cost method chooses variables with small shipping costs to be basic variables, you might think that this method would always yield a basic feasible solution with a relatively low total shipping cost.

Winston, W.L. (2004:380), *Vogel's method for finding a basic feasible solution begins by computing for each row (and column) a "penalty"equal to the difference between the two* smallest costs in the row (column). Next find the row or column with the largest penalty. Choose as the first basic variable the variable in this row or column that has the smallest shipping cost. As described in the Northwest-Corner and Minimum-Cost methods, make this variable as large as possible, cross out a row or column, and change the supply or demand associated with the basic variable. Now recompute penalties (using only cells that do not lie in a crossed-out row or column), and repeat the procedure until only one uncrossed cell remains. Set this variable equal to the supply or demand associated with the variable, and cross *out the variable's row and column. A basic* feasible solution has now been obtained.

Winston, W.L. (2004:382), of the three methods we have discussed for finding a basic feasible solution, the Northwest-*Corner method requires the least effort, and Vogel's* method requires the most effort. Extensive research (Glover et al. (1974)) has shown, *however, that when Vogel's method is used to find an initial basic feasible solution, it* usually takes subtantially fewer pivots than if the other two methods had been used. For this reason, the Northwest-Corner and Minimum-Cost methods are rarely used to find a basic feasible solution to a large transportation problem.

Anderson, D.R., Sweeney, D.J., Williams, T.A., Martin, K. (2008:436), in generalizations of the assignment problem where one agent can be assigned to two or more tasks. Thus, we see that one advantage of formulating and solving assignment problems as linear programs is that special cases such as the situation involving multiple assignments can be easily handled.

Stevenson, W.J., Ozgur, C., (2007:303), summary of the transportation method:

(1) Obtain an initial solution. Use either the Northwest-Corner method, the Minimum-*Cost method (the Intuitive method), or the Vogel's approximation* method. Generally, the Minimum-*Cost method and Vogel's approximation* method are the preferred approaches.
(2) Evaluate the solution to determine if it is optimal. Use either the Stepping-Stone method or MODI. The solution is not optimal if any unoccupied cell has a negative cell evaluation.
(3) If the solution is not optimal, select the cell that has the most negative cell evaluation. Obtain an improved solution using the Stepping-Stone method.
(4) Repeat steps (2) and (3) until no cell evaluations (reduced costs) are negative. Once you have identified the optimal solution, compute its total cost.
(5) Special issues, determining if there are alternate optimal solutions.

Problem transportasi, dari tahun ke tahun perkuliahan biasanya diselesaikan dengan cara perhitungan VAM and MODI, Northwest-Corner rule and Stepping-Stone, dan Minimum-Cost method akan tetapi biasanya pada saat penyelesaian ini kita lupa atau tidak terdeteksi suatu problem dengan 2 (dua) atau bahkan 3 (tiga) jawaban penyelesaian optimal (multiple optimal solutions) walaupun sudah menggunakan teori yang sangat rumit dan kompleks.

Pada kesempatan ini, penulis ingin menyampaikan penyelesaian multiple optimal solutions dengan menggunakan metode VAM and MODI. Yang dimaksud dengan multiple optimal solutions adalah penyelesaian suatu problem transportasi dengan jawaban hasil optimal yang lebih dari 1 (satu) jawaban, bisa 2 (dua) jawaban atau bahkan 3 (tiga) jawaban, dan dengan total biaya minimal/optimal yang sama besarnya untuk tiap jawaban.

Problem Transportasi:

	Destination (Cost/unit)			
Source	1	2	3	Supply
A	$15	$14	$20	800
B	$14	$13	$18	900
C	$4	$6	$8	1100
Demand	1300	350	1150	2800/2800

VAM and MODI:

1	Destination (Cost/unit)				Opportunity Cost		
Source	1	2	3	Supply	1	2	3
A	$15	$14	$20	800	1	1	1
B	$14	$13	$18	900	1	1	
C	$4	$6	$8	1100	2		
Demand	1300	350	1150	2800/2800			
1	10	7	10				
2	1	1	2				
3							

*Choose the highest opportunity cost with the lowest cost, i.e. $4, therefore we choose the highest opportunity cost = 10 at column 1.
*Row 3 eliminated, since all 1100 has been transfered to C1.
*Choose the highest opportunity cost = 2 at column 3.
*Row 2 eliminated, since all 900 has been transfered to B3.

2	Destination (Cost/unit)			
Source	1(K1)	2(K2)	3(K3)	Supply
A(R1)	200	350	250	
B(R2)			900	
C(R3)	1100			
Demand	1300	350	1150	

R1 + K3 = 20, letting R1 = 0, K3 = 20
R2 + K3 = 18, R2 + 20 = 18, R2 = -2
R1 + K2 = 14, 0 + K2 = 14, K2 = 14
R1 + K1 = 15, 0 + K1 = 15, K1 = 15
R3 + K1 = 4, R3 + 15 = 4, R3 = -11

Improvement Index: Iij = Cij - Ri – Kj
I21 = C21 - R2 - K1 = 14 - (-2) - 15 = +1
I22 = C22 - R2 - K2 = 13 - (-2) - 14 = +1
I32 = C32 - R3 - K2 = 6 - (-11) - 14 = +3
I33 = C33 - R3 - K3 = 8 - (-11) - 20 = -1

*Since the unoccupied cell I33 = -1 (negative), we therefore should select the smallest number found in those squares on the closed path containing minus sign.
*The smallest number with the minus sign is 250, therefore we should assign 250 at cell I33 with -1 (negative).
*250 will be subtracted from each cell with a minus sign and added to each cell with a plus sign.

3	Destination (Cost/unit)			
Source	1(K1)	2(K2)	3(K3)	Supply
A(R1)	+200	350	250-	
B(R2)			900	
C(R3)	-1100		+	
Demand	1300	350	1150	

Option 1

4	Destination (Cost/unit)			
Source	1	2	3	Supply
A B C	450 850	350	 900 250	
Demand	1300	350	1150	

R3 + K3 = 8, letting R3 = 0, K3 = 8
R2 + K3 = 18, R2 + 8 = 18, R2 = 10
R3 + K1 = 4, 0 + K1 = 4, K1 = 4
R1 + K1 = 15, R1 + 4 = 15, R1 = 11
R1 + K2 = 14, 11 + K2 = 14, K2 = 3

Improvement Index: Iij = Cij - Ri – Kj
I13 = C13 - R1 - K3 = 20 - 11 - 8 = +1
I21 = C21 - R2 - K1 = 14 - 10 - 4 = 0
I22 = C22 - R2 - K2 = 13 - 10 - 3 = 0
I32 = C32 - R3 - K2 = 6 - 0 - 3 = +3

Total Minimal Cost for Option 1

A1 = 450 x $15 = $ 6,750
A2 = 350 x $14 = $ 4,900
B3 = 900 x $18 = $16,200
C1 = 850 x $4 = $ 3,400
C3 = 250 x $8 = $ 2,000

Total Cost for Option 1 = $33,250

*Since all the unoccupied cell is greater than or equal to zero, an optimum solution has been reached! (all are positive or zero)
*Because I21 and I22 are equal to zero, we note that there is additional 2 (two) or multiple optimal solutions exist.
*The initial solution found was optimal, but the improvement index for two of the empty cells were zero, indicating 2 (two) other optimal solutions. Use a stepping-stone path to develop these 2 (two) other optimal solutions, starts at I21 and I22.

5	Destination (Cost/unit)			
Source	1(K1)	2(K2)	3(K3)	Supply
A(R1)	450	350		
B(R2)	+ ←		900 - ↑	
C(R3)	↓ - 850 →		250 +	
Demand	1300	350	1150	

*Use a stepping-stone path, we see that the lowest number of units in a cell where a subtraction is to be made is 850 units.
*Therefore, 850 units will be subtracted from each cell with a minus sign and added to each cell with a plus sign.
*The result (starts at I21) is:

Option 2

6	Destination (Cost/unit)			
Source	1	2	3	Supply
A	450	350		
B	850		50	
C			1100	
Demand	1300	350	1150	

Total Minimal Cost for Option 2
A1 = 450 x $15 = $ 6,750
A2 = 350 x $14 = $ 4,900
B1 = 850 x $14 = $11,900
B3 = 50 x $18 = $ 900
C3 = 1100 x $8 = $ 8,800
Total Cost for Option 2 = $33,250

7	Destination (Cost/unit)			
Source	1(K1)	2(K2)	3(K3)	Supply
A(R1)	+450	-350		
B(R2)		+	-900	
C(R3)	-850		+250	
Demand	1300	350	1150	

*Use a stepping-stone path, we see that the lowest number of units in a cell where a subtraction is to be made is 350 units.
*Therefore, 350 units will be subtracted from each cell with a minus sign and added to each cell with a plus sign.
*The result (starts at I22) is:

Option 3

8	Destination (Cost/unit)			
Source	1	2	3	Supply
A	800			
B		350	550	
C	500		600	
Demand	1300	350	1150	

Total Minimal Cost for Option 3
A1 = 800 x $15 = $ 12,000
B2 = 350 x $13 = $ 4,550
B3 = 550 x $18 = $ 9,900
C1 = 500 x $4 = $ 2,000
C3 = 600 x $8 = $ 4,800
Total Cost for Option 3 = $33,250

KESIMPULAN

Terdapat 3 (tiga) jawaban optimal dalam penyelesaian problem transportasi di atas dengan VAM and MODI method. Ke 3 (tiga) jawaban penyelesaian optimal menunjukkan alokasi yang berbeda akan tetapi menghasilkan total biaya minimal yang sama besarnya baik untuk alokasi yang terbentuk pada option 1, untuk alokasi yang terbentuk pada option 2, maupun untuk alokasi yang terbentuk pada option 3.

The special cases of transportation problem, i.e. multiple optimal solutions, dapat diselesaikan dengan lebih cepat dengan VAM and MODI method, yang disingkat dengan VAM.

VAM dan MODI walaupun tidak selalu menghasilkan pemecahan optimum akan tetapi bisa juga memberikan pemecahan yang optimum. VAM tidak menjamin suatu penyelesaian yang optimum, akan tetapi sangat berguna karena alasan berikut ini: (1) sering menghasilkan pemecahan optimum, (2) dapat menghasilkan penyelesaian yang mendekati optimum dengan usaha yang tidak banyak, sehingga dapat dipergunakan untuk melangkah menuju ke pemecahan yang optimal.

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