Seasonal Inventory Decisions (Single-Period Inventory Models)

Rudy Santosa Sudirga

Penulis

Rudy Santosa Sudirga, adalah dosen di program studi manajemen, Universitas Bunda Mulia, Jakarta, dalam bidang kuantitatif.

Abstract

One of the dilemmas facing many retailers is how to handle seasonal goods, such as sarung dress or batik dress, biscuits and syrups during Idul Fitri celebration. Often they cannot be sold at full markup next year because of changes in styles, and expiry dates for foods and beverages. Furthermore, the lead time can be longer than the selling season, allowing no second chance to rush through another order to cover unexpectedly high demand. At the end of the period the product has either sold out or there is a surplus of unsold items to sell for a salvage value. The single-period inventory models are used in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods.

This type of situation is often called the newsboy problem. If the newspaper seller does not buy enough newspapers to resell on the street corner, sales opportunities are lost. If the seller buys too many newspapers, the overage cannot be sold because nobody wants yesterday's newspaper.

At this opportunity, we will use the single-period inventory models decisions to create an optimal order quantity decision with the highest expected payoff. We will be able to use this decision process for all such items over many selling seasons to maximize profits.

However, it is not a foolproof, and it can result in an occasional bad outcome. It will depend on the situation, if the probability of demand follows the demand probability planning, then you will be able to get the best decision to prepare and execute an order.

Kata kunci:

*Seasonal Inventory, Single-Period Inventory Models, Probability of Demand***.**

PENDAHULUAN

Seasonal Inventory decisions atau dikenal dengan *Single-Period Inventory Models decisions* atau disebut pula dengan *newsboy problem* (*if the newspaper seller does not buy enough newspapers to resell on the street corner*, *sales opportunities are lost)* merupakan suatu problem yang sangat sulit dilaksanakan pengambilan keputusan *inventory* nya karena keadaan yang sangat sulit untuk diantisipasi atau diramalkan. Selain itu produk yang disiapkan untuk *inventory* tidak dapat atau susah untuk dijual kembali pada hari berikutnya atau bila waktu musimnya sudah lewat. Misalnya penjualan pohon natal di hari raya natal, yang mana penjual pohon natal akan sulit menjual pohon natal bila musim perayaan natal sudah lewat. Hal ini terkait pula dengan keputusan persiapan *inventory* yang harus kita ambil, apakah akan mengambil keputusan persiapan *inventory* yang banyak dan berapa jumlahnya, atau ingin mengambil keputusan persiapan *inventory* yang sedikit akan tetapi berdampak akan mengalami kehilangan penjualan bila ternyata nanti permintaan pembelian dari pelanggan meningkat dibandingkan pengalaman sebelumnya, misalnya pengalaman tahun lalu, kemarin, dan lain-lain.

Untuk itu di dalam banyak buku manajemen operasional penyelesaian masalah dan problem ini juga banyak variasi penyelesaiannya dan buku yang satu dengan buku yang lainnya menampilkan penyelesaian yang berbeda atau hampir mirip dan satu sama lain saling tidak sama penyelesaiannya. Misalnya ada buku yang menyelesaikan keputusan *inventory* harus dengan *probabilistic model*, ada pula yang menyelesaikan keputusan inventory dengan pendekatan *service level* dan *normal distribution*, dan bahkan ada satu buku yang menyelesaikan dengan dua cara yaitu menghitung dulu *probability* (*service level*) lalu diselesaikan dengan cara *normal distribution* dan *discrete distribution*.

Dalam tulisan ini, penulis bermaksud menunjukkan penyelesaian dengan cara *probabilistic model* dan dengan memperhatikan keuntungan (*payoff*) yang didapatkan, dan hasilnya dibandingkan dengan penyelesaian yang dilakukan dengan *software QM for Windows.* Maksud penulisan ini adalah bahwa penyelesaian dengan cara ini akan mendapatkan hasil yang paling mendekati kebenaran dan dalam hal ini penulis ingin memperlihatkannya pada beberapa contoh penyelesaian di bawah ini.

Berikut penulis kutipkan teori penyelesaian dari beberapa buku manajemen operasional, manajemen sains, dan *quantitative analysis for management* secara singkat saja, yaitu sebagai berikut:

Heizer and Render (2014:530-531), all the inventory models we have discussed so far make the asumption that the demand for a product is constant and certain. We now relax this assumption. The following inventory models apply when product demand is not known but can be specified by means of a probability distribution. These types of models are called probabilistic models. Probabilistic models are real-world adjusment because demand and lead time won't always be known and constant.

An important concern of management is maintaining and adequate service level in the face of uncertain demand. The service level is the complement of the probability of a stockout. For instance, if the probability of a stockout is 0.05, then the service level is 0.95. Uncertain demand raises the possibility of a stockout. One method of reducing stockouts is to hold extra units in inventory. $_s$As we noted earlier such inventory is referred to as safety stock. Safety stock involves adding a number of units as a buffer to the reorder point.

Heizer and Render (2014:536), a single-period inventory model describes a situation in which one order is placed for a product. At the end of the sales period, any remaining product has little or no value. This is a typical problem for Christmas trees, seasonal goods, bakery goods, newspapers, and magazines. (Indeed, this inventory issue is often called the "newstand problem"). In other words, even though items at a newstand are ordered weekly or daily, they cannot be held over and used as inventory in the next sales period. So our decision is how much to order at the beginning of the period.

Because the exact demand for such seasonal products is never known, we consider a probability distribution related to demand. If the normal distribution is assumed, and we stocked and sold an average (mean) of 100 Christmas trees each season, then there is a 50% chance we would stock out and a 50% chance we would have trees left over. The determine the optimal stocking policy for trees before season begins, we also need to know the standard deviation and consider the cost of shortage (C_s) = sales price per unit – cost per unit, and the cost of overage (C_o) = cost per unit – salvage value per unit.

The service level, that is the probability of not stocking out, is set at:

Service level = $\frac{Cs}{Cs+Co}$ *, thereforewe should consider increasing our order quantity until the service level is equal to or more than the ratio of* $\frac{Cs}{Cs+Co}$*. We need to find the Z scorefor this normal distribution that yields a probability of* $\frac{Cs}{Cs+Co}$*. From the formula of the normal distribution,* $Z = \frac{X-\mu}{\sigma}$*, then the optimal stocking level (order quantity) is equal to* $X = \mu + Z\sigma$*.*

Evans and Collier (2007:522-523), the single-period inventory model applies to inventory situations in which one order is placed for a good in antcipation of a future selling sesaon where demand is uncertain. At the end of the period the product has either sold out or there is a surplus of unsold items to sell for a salvage value. Single-period models are used in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods. In such a single-period inventory situation, the only inventory decision is how much of the product to order at the start of the period. Because newspaper sales are a typical example of the single-period situation, the single-period inventory problem is sometimes referred to as the newsvendor problem. The newsvendor problem can be solved using a technique called marginal economic analysis, which compares the cost or loss of ordering one additional item with the cost or loss of not ordering one additional item. The costs involved are defined as salvage cost (C_s*) = the cost per item of overestimating demand, this cost represents the loss of ordering one additional item and finding that it cannot be sold; and the shortage cost (*C_u*) = the cost per item of underestimating demand, this cost represents the opportunity loss of not ordering one additional item and finding that it could have been sold.*

The optimal order quantity is the value of Q that satisfies:

P *(demand* $\leq Q$*) =* $\frac{Cu}{Cu+Cs}$ *, locate P on the normal distribution table and find the associated Z-value, and then solve the optimal order quantity by using the formula of the normal distribution,* $Z = \frac{Q-\mu}{\sigma}$*, which is equal to* $Q = \mu + Z\sigma$*.*

Jacobs, Chase, Aquilano (2009:551), a simple way to think about a single-period inventory model is to consider how much risk we are willing to take for running out of inventory. Let's consider that the newsperson selling newspapers in the sales stand had collected data over *a few months and had found that on average each Monday 90 newspapers were sold with a standard deviation of 10 newspapers (of course, this assumes that the newspapers had never run out).*

To make this more useful, it would be good to actually consider the potential profit and loss associated with stocking either too many or too few newspapers on the stand.

Russell and Taylor (2011:574-575), there are several ways to determine the amount of the safety stock. One popular method is to establish a safety stock that will meet a specified service level. The service level is the probability that the amount of inventory on hand during the lead time is sufficient to meet expected demand, that is, the probability that a stockout will not occur. The term service is used, since the higher the probability that inventory will be on hand, the more likely that customer demand will be met, that is, that the customer can be served. A service level of 90% menas that there is a 0.90 probabiliy that demand will be met during the lead time, and the probability that a stockout will occur is 10%. The sevice level is typically a policy decision based on a number of factors, including carrying costs for the extra safety stock and lost sales if customer demand cannot be met.

Stevenson (2009:581-582), the single-period model (sometimes referred to as newsboy problem) is used to handle ordering of perishables (fresh fruits, vegetables, seafood, cut flowers) and items that have a limited useful life (newspapers, magazines, spare parts for specialized equipment). The period for spare parts is the life of the equipment, assuming that the parts cannot be used for other equipment. What sets unsold or unused goods apart is that they are not typically carried over from one period to the next, at least not without penalty. Day-old baked goods, for instance, are often sold at reduced prices, leftover seafood may be discarded, and out-of-date magazines may be offered to used book stores at bargain rates. There may even be some cost associated with disposal of leftover goods.

Analysis of single-period situations generally focuses on two costs, shortage and excess. Shortage cost may include a charge for loss of customer goodwill as well as the opportunity cost of lost sales. Generally, shoratge cost is simply unrealized profit per unit.

The concept of identifying an optimal stocking level is perhaps easiest to visualize when demand is uniform. Choosing the stocking level is similar to balancing a seesaw, but instead of a *person on each end of the seesaw, we have excess cost per unit. On one end of the distribution and shortage cost per unit on the other. The optimal stocking level is analogous to the fulcrum of the seesaw, the stocking level equalizes the cost weights. The service level is the probability that the demand will not exceed the stocking level, and computation of the service level is the key to determining the optimal stocking level.*

Render, Stair, Hanna, Hale (2015:229-230), there are some products for which a decision to meet the demand for a single time period is made, and items that do not sell during this time period are of no value or have a greatly reduced value in the future. For example, a daily newspaper is worthless after the next papaer is available. Other examples include weekly magazines, programs printed for athletic events, certain prepared foods that have a short life, and some seasonal clothes that have greatly reduced value at the end of the season. This type of problem is often called the news vendor problem or a single-period inventory model.

A decision-making approach using marginal profit and marginal loss is called marginal analysis. Marginal Profit (MP) is the additional profit achieved if one additional unit is stocked and sold. Marginal Loss (ML) is the loss that occurs when an additional unit is stocked but cannot be sold.

Render, Stair, Hanna, Hale (2015:232), Steps of Marginal Analysis with the Normal Distribution:

1. *Determine the value of* $P = \frac{ML}{ML + MP}$ *for the problem*
2. *Locate P on the normal distribution table and find the associated Z-value*
3. *Find X using the relationship* $Z = \frac{X - \mu}{\sigma}$
4. *To Solve the resulting stocking policy:* $X = \mu + Z\sigma$

Jadi pada problem *single-period inventory models* dapat diperolah jawaban yang paling optimal adalah apabila kita melakukan *order quantity* seperti pada *Steps of Marginal Analysis with the Normal Distribution* yang dipaparkan di atas dan hal ini merupakan penyelesaian di banyak buku *Operations Management* dan *Management Science*.

Render, Stair, Hanna, Hale (2015:232), Steps of Marginal Analysis with Discrete Distribution:

1. *Determine the value of P =* $\frac{ML}{ML + MP}$ *for the problem*
2. *Construct a probability table and add a cumulative probability column*
3. *Keep ordering inventory as long as the probability (P) of selling at least one additional unit is greater than* $\frac{ML}{ML + MP}$

Krajewski, Ritzman, Malhotra (2013:370-371), for one period decisions (seasonal inventory decisions) or single-period inventory models the following process is a straightforward way to analyze such problems and decide on the best order quantity:

1. *List the different levels of demand that are possible, along with the estimated probability of each.*
2. *Develop a payoff table that shows the profit for each purchase quantity, Q, at each assumed demand level, D. Each row in the table represents a different order quantity, and each column represents a different demand level. The payoff for a given quantity-demand combination depends on whether all units are sold at the regular profit margin during the regular season, which results in two possible cases.*
   a. *If demand is high enough (Q ≤ D), then all units are sold at the full profit margin, p, during the regular season, Payoff = (Profit per unit)(Purchase quantity) = pQ*
   b. *If the purchase quantity exceeds the eventual demand (Q > D), only D units are sold at the full profit margin, and the remaining units purchased must be disposed of at a loss, l, after the season. In this case, Payoff = (Profit per unit sold during season)(Demand) – (Loss per unit)(Amount disposed of after season) = pD – l(Q – D)*
3. *Calculate the expected payoff for each Q (or row in the payoff table) by using the expected value decision rule. For a specific Q, first multiply each payoff in the row by the demand probability associated with the payoff, and then add these products.*
4. *Choose the order quantity Q with the highest expected payoff.*

PROBLEM

Suatu toko ritel menjual produk *ornament* dengan profit \$10 per unit selama musim perayaan, akan tetapi toko akan menjual rugi \$5 per unit setelah musim perayaan selesai. Berikut adalah *discrete probability distribution* selama musim perayaan yang telah berhasil diidentifikasi:

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

Berapa banyak *ornament* yang seharusnya di order atau disediakan oleh toko tersebut?

Jawaban

1. Jika menggunakan cara *Marginal Analysis with the Normal Distribution* maka didapatkan MP = \$10 dan ML = \$5

$P = \frac{ML}{ML + MP} = \frac{\$5}{\$10+\$5} = 0.33$

Since the normal table has cumulative areas under the curve between the left side and any point, we look for 0.67 (= 1.0 – 0.33) in order to get the corresponding Z value.

Z value = 0.44. In this problem, $\mu = 30$ *and* $\sigma = 15.81$

$Z = \frac{X - \mu}{\sigma}$

$X = \mu + Z\sigma = 30 + 0.44(15.81) = 37$

Jadi keputusannya bahwa kita harus melakukan ***order*** atau melakukan ***inventory*** sebesar **37 unit**, dan hal ini merupakan penyelesaian di banyak buku *Operations Management* dan *Management Science.*

2. Jika menggunakan cara *Marginal Analysis with Discrete Distribution*

$P \geq \frac{ML}{ML + MP} = \frac{\$5}{\$10+\$5} = 0.33$

Jadi keputusan melakukan inventory adalah bila $P \geq 0.33$

Berikutnya kita membuat tabel dengan menambahkan *cumulative probability column* untuk melihat kemungkinan demand yang memenuhi syarat $P \geq 0.33$

Demand	Demand Probability	Cumulative Probability (Probability that demand will be at this level or greater)
10	0.2	1.0
20	0.3	0.8
30	0.3	0.5
40	0.1	0.2
50	0.1	0.1

Dari tabel ini dapat dilihat bahwa *cumulative probability* yang memenuhi syarat P $\geq$ 0.33 adalah pada *demand* 30, 20 atau 10, akan tetapi kelemahan metode ini adalah kita tidak dapat menentukan secara pasti berapakah yang harus kita *order* apakah 30, 20 atau 10? Sebab jika kita melakukan order 30 *ornament*, maka di dapatkan bahwa *marginal profits* masih lebih besar daripada *marginal loss* karena P pada 30 unit = 0.5 > 0.33

3. Jika menggunakan cara dengan menentukan *payoff* (keuntungan)

Setiap jumlah *demand* adalah kandidat untuk mendapatkan *order quantity* yang paling baik, sehingga *payoff* tabel akan mempunyai 5 baris sesuai dengan banyaknya demand yaitu 5. Profit = p = $10 per unit

Lalu kita melakukan *trial order* Q untuk setiap jumlah *demand* D dengan anggapan bahwa *order* Q akan habis terjual atau $Q \leq D$, maka dipakai formula:

Untuk Q = 10 dan D = 10, Payoff = pQ = ($10)(10) = $100,

Untuk Q = 10 dan D = 20, Payoff = pQ = ($10)(10) = $100

Untuk Q = 40 dan D = 40, Payoff = pQ = ($10)(40) = $400

Untuk Q = 40 dan D = 50, Payoff = pQ = ($10)(40) = $400, demikian seterusnya

Lalu kita melakukan *trial order* Q untuk setiap jumlah *demand* D dengan anggapan bahwa *order* Q tidak akan habis terjual atau $Q > D$, maka dipakai formula:

Untuk Q = 20 dan D = 10, Payoff = $pD – l(Q – D)$ = ($10)(10) – ($5)(20 – 10) = $50

Untuk Q = 30 dan D = 20, Payoff = $pD – l(Q – D)$ = ($10)(20) – ($5)(30 – 20) = $150

Untuk Q = 40 dan D = 30, Payoff = $pD – l(Q – D)$ = ($10)(30) – ($5)(40 – 30) = $250

Untuk Q = 50 dan D = 40, Payoff = $pD – l(Q – D)$ = ($10)(40) – ($5)(50 – 40) = $350,

demikian seterusnya, sehingga kita dapat membuat tabel *payoff* sebagai berikut:

		Demand				
		10	20	30	40	50
Oder Quantity (Q)	10	100	100	100	100	100
	20	50	200	200	200	200
	30	0	150	300	300	300
	40	-50	100	250	400	400
	50	-100	50	200	350	500

Selanjutnya kita dapat menghitung *expected payoff* untuk setiap *order quantity* (Q) dan untuk setiap jumlah *demand* dengan cara mengalikan setiap *demand probability* dengan setiap *payoff* sebagai berikut:

Q = 10, Payoff = 0.2($100) + 0.3($100) + 0.3($100) + 0.1($100) + 0.1($100) = $100

Q = 20, Payoff = 0.2($50) + 0.3($200) + 0.3($200) + 0.1($200) + 0.1($200) = $170

Q = 30, Payoff = 0.2($0) + 0.3($150) + 0.3($300) + 0.1($300) + 0.1($300) = $195

Q = 40, Payoff = 0.2(-$50) + 0.3($100) + 0.3($250) + 0.1($400) + 0.1($400) = $175

Q = 50, Payoff = 0.2(-$100) + 0.3($50) + 0.3($200) + 0.1($350) + 0.1($500) = $140

Dari tabel *payoff* terlihat bahwa *expected payoff* yang **terbesar** yaitu **$195** adalah pada saat kita melakukan **order Q = 30 unit**.

4. Jika menggunakan penyelesaian dengan software QM for Windows

Parameter	Value		
	10	xxxxxxx	
Marginal cost per unit	5	xxxxxxx	
Cutoff ML/(ML+MP)	0.3333333		
Probability Distribution	DEMAND	PROBABILITY	Prob sales at this level or higher
Demand 1	10	0.2	1 >= .3333333
Demand 2	20	0.3	.8 >= .3333333
Demand 3	30	0.3	.5 >= .3333333
Demand 4	40	0.1	0.2
Demand 5	50	0.1	0.1
Optimal Order Quantity	30		

Dengan penyelesaian *software* QM for Windows, didapatkan bahwa ***optimal order quantity* = 30 unit**.

KESIMPULAN:

Penulis berkesimpulan bahwa:

- Penyelesaian dengan cara 1 (*marginal analysis with the normal distribution*) akan memberikan hasil yang kurang tepat karena mengabaikan *probability* yang terjadi pada *demand* serta *profits* yang akan diperoleh oleh pengambil keputusan. Dan cara ini merupakan penyelesaian di banyak buku *Operations Management* dan *Management Science.*

- Penyelesaian dengan cara 2 (*marginal analysis with discrete distribution*) akan memberikan banyak *output* jawaban sehingga pengambil keputusan harus melakukan pilihan order yaitu apakah 30, 20 atau 10 dan cara ini akan memberikan banyak opsi jawaban untuk memutuskan *optimal order quantity* karena opsi ini memakai dasar boleh melakukan jumlah *order* jika jumlah hasil probability tersebut (*cumulative probability*), yaitu $P \geq \frac{ML}{ML + MP}$, yang mana akan memberikan jawaban yang lebih dari satu jawaban seperti terlihat pada penyelesaian di atas.

- Penyelesaian dengan cara 3 yaitu menggunakan cara dengan menentukan *payoff* (keuntungan), menurut penulis adalah cara yang lebih mudah dan lebih tepat karena selain memperhatikan *payoff* (keuntungan) yang akan didapatkan oleh pengambil keputusan untuk melaksanakan jumlah *optimal order quantity*, juga memperhatikan *probability* yang terjadi pada permintaan (*demand*) saat berlangsungnya *seasonal celebration*.

Selain itu hasil cara dengan menentukan *payoff* (keuntungan) ini juga memberikan hasil jawaban yang sama dengan jawaban menggunakan cara 4 dengan *software* QM for Windows, yaitu mengisyaratkan agar pengambil keputusan melaksanakan *optimal order quantity* sebesar 30 unit.

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